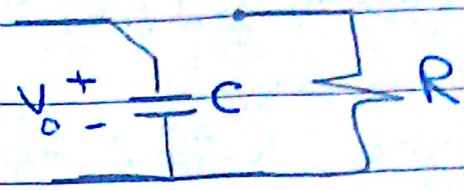
Date:

Subject: Lec 4

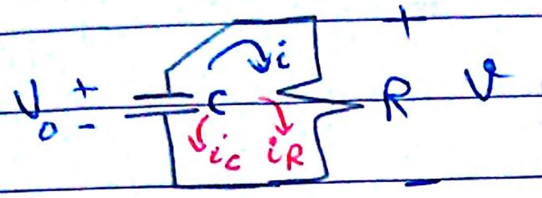
* Natural Response of RC Circuits

initial Condition ⇐

⇐ قد يكون موجود او غير موجود

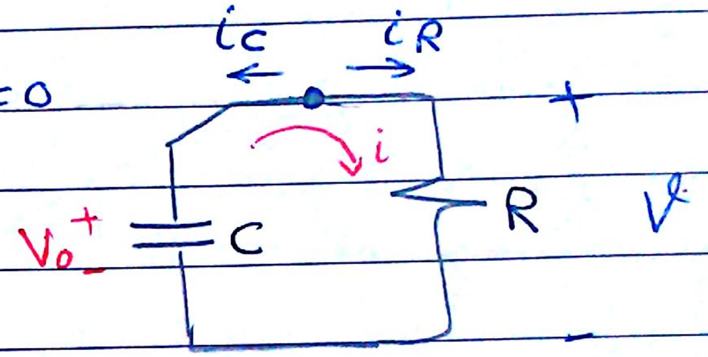


response بدأ ال ← $t=0$



$i_R + i_C = 0$ التيارين خارجين طبقاً لقانون كيرشوف

$t=0$



$$i_C + i_R = 0$$

$$C\frac{dv}{dt} + \frac{v}{R} = 0$$

$$\boxed{\frac{dv}{dt} + \frac{1}{CR}v = 0}$$

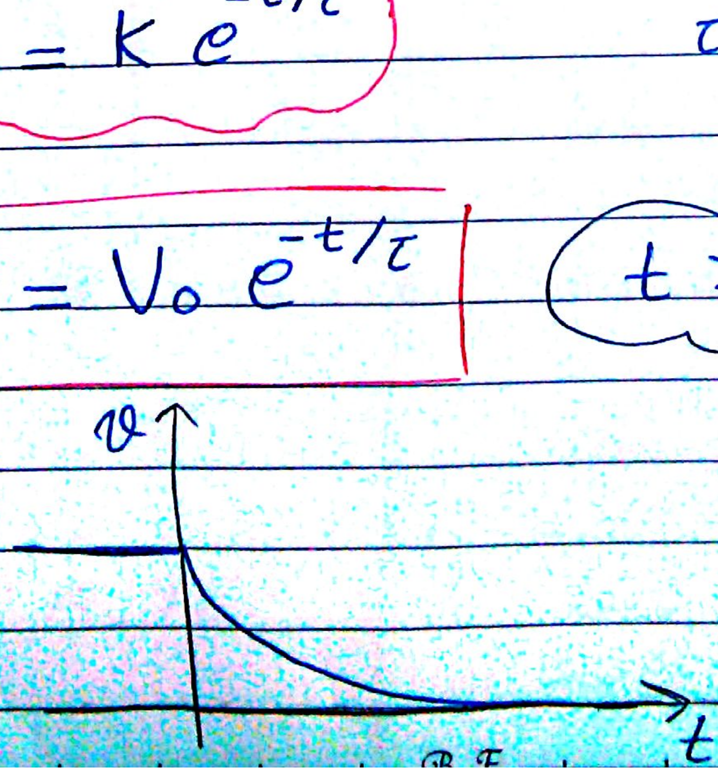
$v(0) = V_0$ at $t=0$

$$v(t) = K e^{-t/\tau}$$

$\tau = CR$

$$\boxed{v(t) = V_0 e^{-t/\tau}}$$

$t \geq 0$



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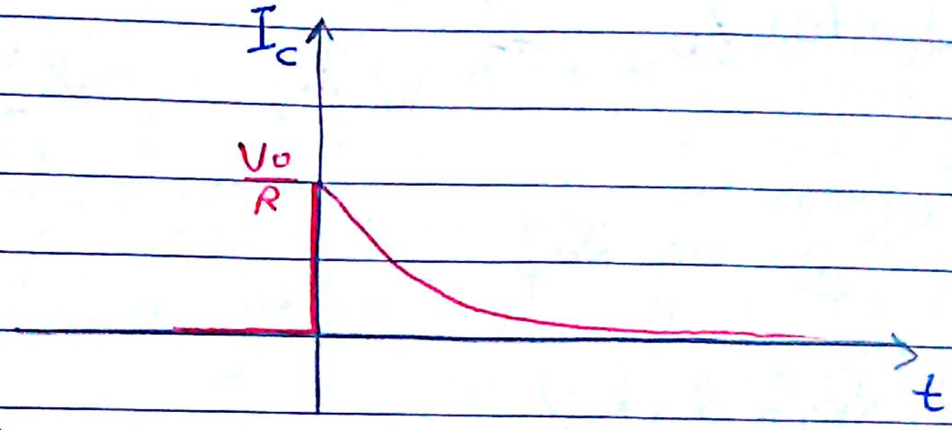
$$i = \frac{v(t)}{R} \quad \text{or} \quad i = C\frac{dV}{dt}$$

$$i(t) = \frac{V_0}{R} e^{-t/\tau} \qquad \boxed{t > 0^+}$$

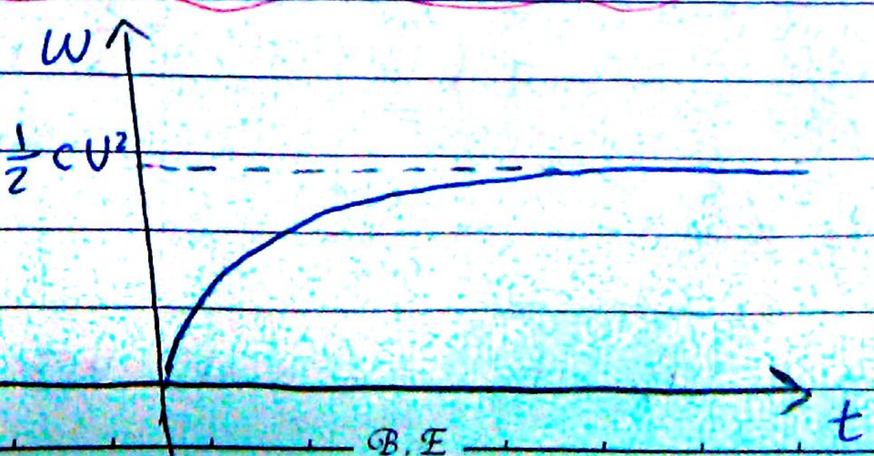


$$P(t) = vi$$

$$P(t) = \frac{V_0^2}{R} e^{-2t/\tau} \qquad \boxed{t > 0^+}$$

$$w(t) = \int_0^t vi\,dt$$

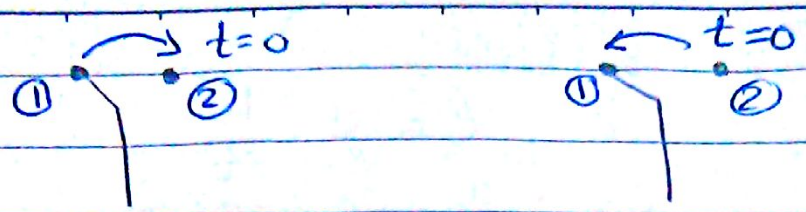
$$w(t) = \frac{1}{2} C V_0^2 \left[1 - e^{-2t/\tau}\right]$$



t

B. E

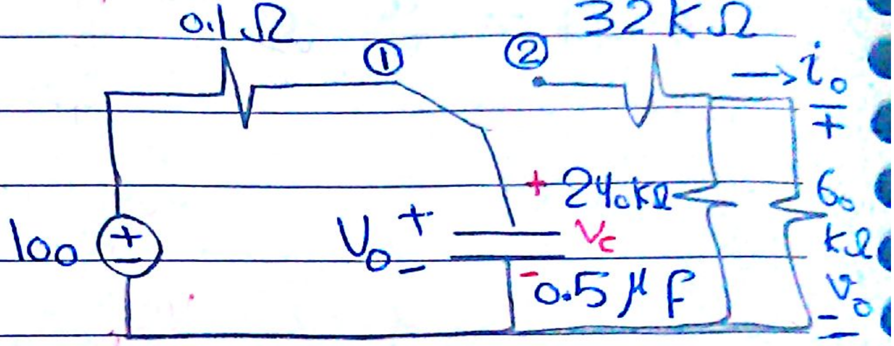
Date: Subject:



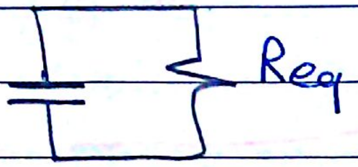
① steady state

المكثف → 0.5

$V_o = 100\ V$



②



$Req = 80\,k\Omega$

$\tau = 80k\Omega * 0.5\,\mu F = 0.04\,S$

$V_c = 100\,e^{-25t} \qquad t \geq 0$

لأن الجهد في المكثف لا يتغير لحظياً

فهو معروف عند $t=0$ & على عكس التيار

$V_o = \frac{48}{80} * 100\,e^{-25t} \qquad t > 0^+$

غير معروف على المقاومة

V_o قبل $t=0$ ⇐ صفر

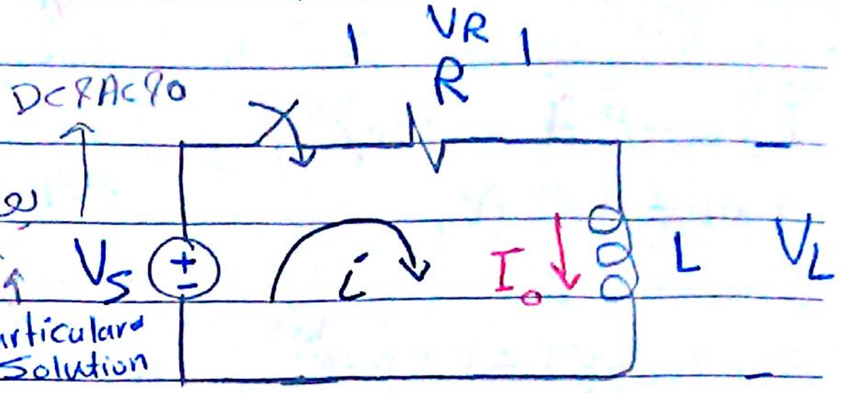
مسموح للجهد يتغير لحظياً على المقاومة

$i_o = \frac{V_o}{60} = 0.001\,e^{-25t} \qquad t > 0^+$

$P_{60k\Omega} = i_o^2 * 60\,k\Omega = \frac{V_o^2}{60k\Omega}$

$W_{60k\Omega} = \int_0^\infty P_{60}\,dt = 0.012\ WS$

⊛ Step Response of RL Circuits "Series"



$$L\frac{di}{dt} + Ri = V_s$$

يعتمد
نوع المصدر

$$i(t) = i_c + i_p \rightarrow \text{Particular Solution}$$

$i_c \rightarrow 0$

Complementary Solution $= i_t + i_{ss} \rightarrow$ Steady State

$\hookrightarrow$ transiant

[illegible]

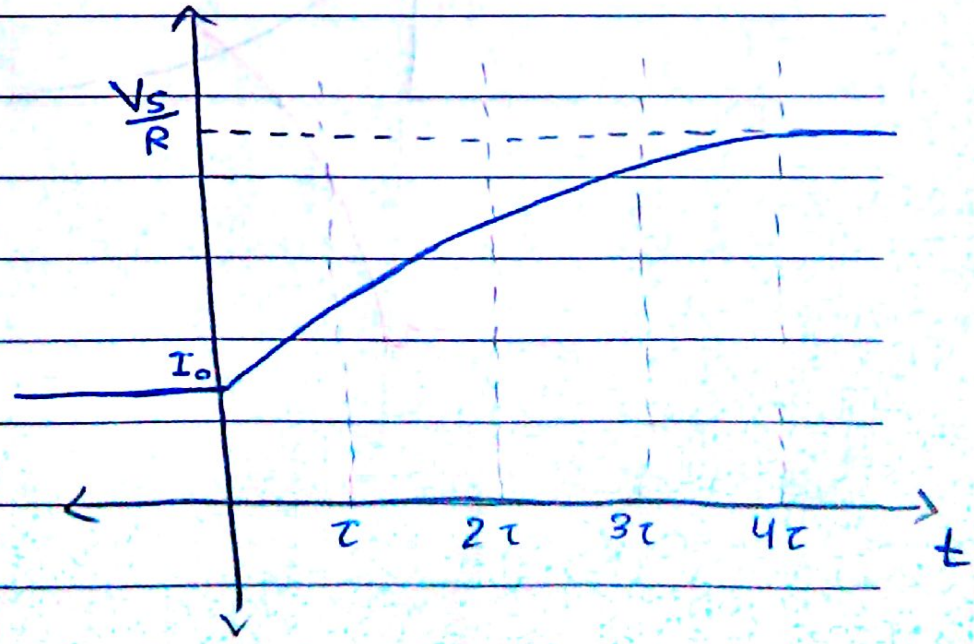
$$i(t) = Ke^{-t/\tau} + i_{ss}$$

$$i(t) = Ke^{-t/\tau} + \frac{V_s}{R}$$

$$i(0) = I_0$$

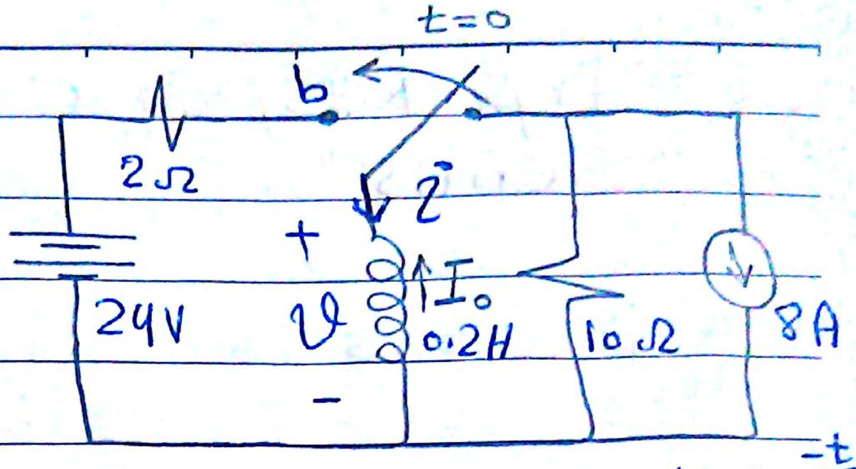
$$\boxed{i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-t/\tau}} \qquad \boxed{t \geq 0}$$

لو $I_0 = \frac{V_s}{R}$
الـ transiant = 0
فهيبدأ الـ
Steady state علطول



Date: Subject:

Ex:



$I_o = -8A = i(0)$

$i(\infty) = 12\ A$

$\tau = 0.1\ s$

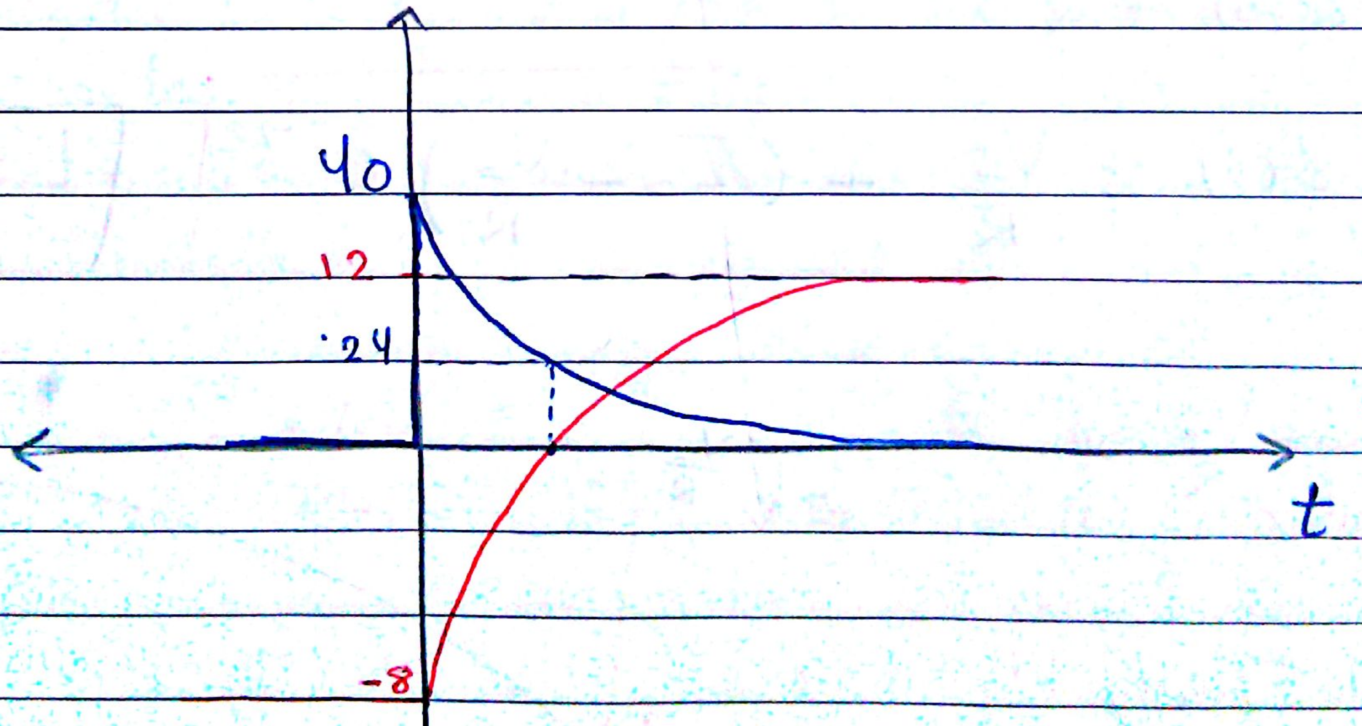
$i = 12 + (-8 - 12)\, e^{-10t}$

$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right) e^{-\frac{t}{\tau}}$

$\boxed{i = 12 - 20\, e^{-10t}}$ $t \geq 0$

$V_L = L \frac{di}{dt}$

$\boxed{v(t) = 40\, e^{-10t}}$ $t > 0^+$



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عند ان لحظة زمنية جهد الملف يساوي جهد المصدر

$\Rightarrow$ لما التيار هيساوي الصفر

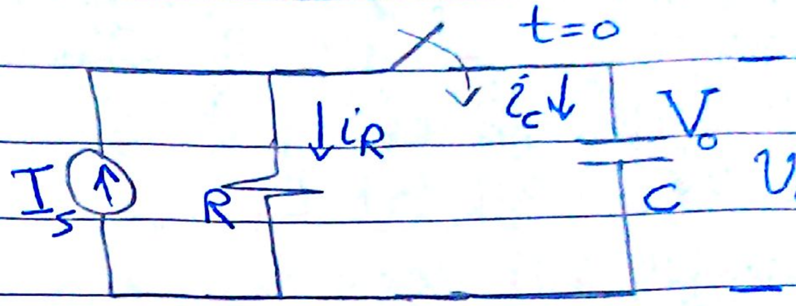
$i(0)$ at $t_1 =$

$V(t) = \underbrace{24}_{\text{جهد المصدر}}$ at t_1.

$t_1 = 51$ ms

Step responce of RC Circuits

$i_C + i_R = I_S$



$C\frac{dv_0}{dt} + \frac{V_0}{R} = I_S$

$(\div C)$

$\frac{dV_0}{dt} + \frac{V_0}{RC} = I_S / C$

$V_0 = \underbrace{I_S R}_{\text{Final SS}} + (\underbrace{V_0}_{IC} - \underbrace{I_S R}_{\text{Final}})e^{-t/\tau}$

initial Voltage ⟷ 0, +ve, -ve

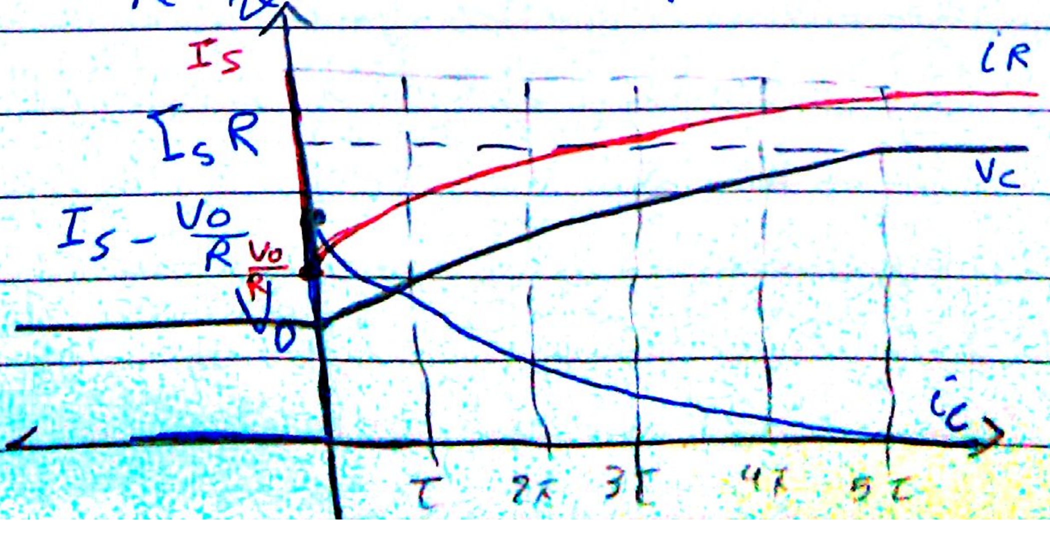
ع المكثف

$\boxed{t \geq 0}$

$\tau = RC$

$i_C = C\frac{dV}{dt} = \left(I_S - \frac{V_0}{R}\right)e^{-t/\tau}$ $\boxed{t > 0^+}$

$i_R = \frac{V}{R} = I_S + \left(\frac{V_0}{R} - I_S\right)e^{-t/\tau}$ $\boxed{t > 0^+}$



لما يوصلها بعد زمن

5 اضعاف ت

Date:

Subject:

في ال ∞ التيار بيؤول الى صفر

تيار المقاومة كان I_s ولحظة قفل المفتاح قل وهايرجع يزيد تاني

$$v = i_R R$$

$$v = (I_s - i_c) R$$

$$= \underbrace{I_s R}_{\text{ثابت}} - i_c R$$

$$\frac{dv}{dt} = -R \frac{di_c}{dt} \qquad (\times c)$$

$$c \frac{dv}{dt} = -cR \frac{di_c}{dt}$$

$$i_c = -cR \frac{di_c}{dt}$$

$$\frac{di_c}{dt} + \frac{1}{RC} i_c = 0$$

$$i_c = K e^{-t/\tau}$$

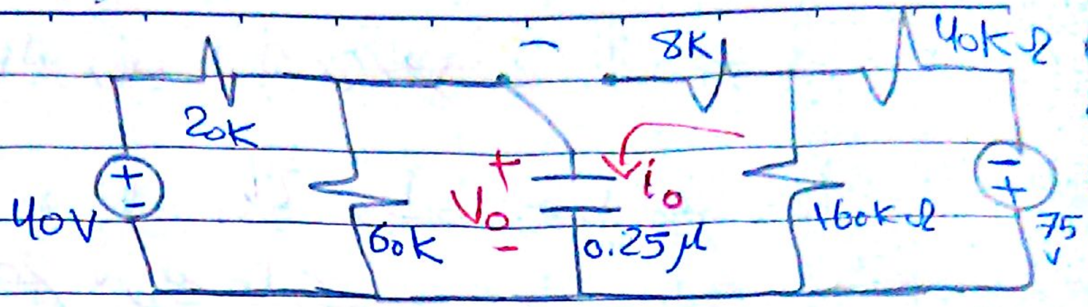
$$v = \frac{1}{c} \int i_c \, dt + V_0$$

$$v = i_R R$$

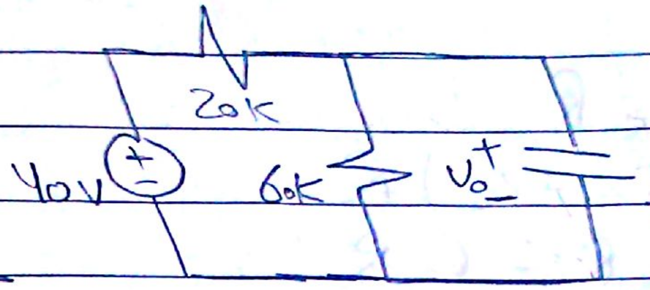
$$= (I_s - i_c) R$$

Date: Subject:

EX:

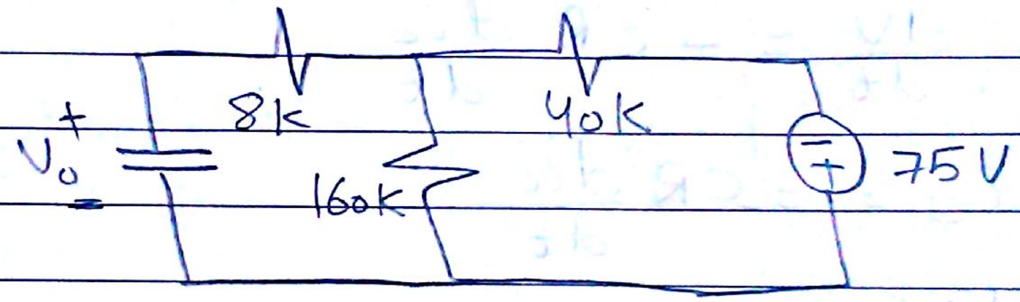


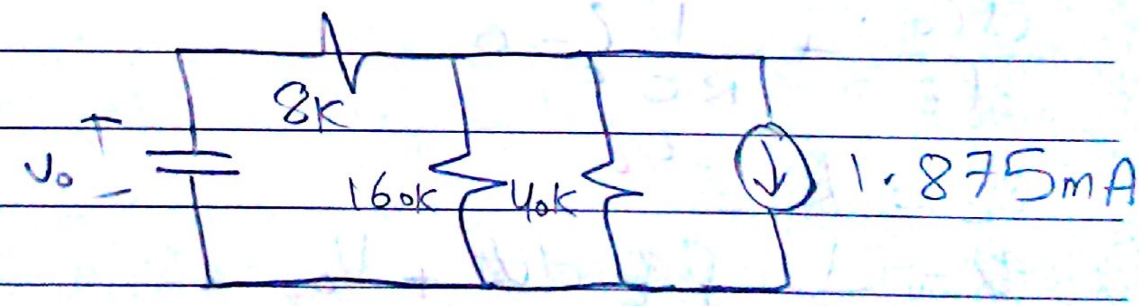
$t < 0$

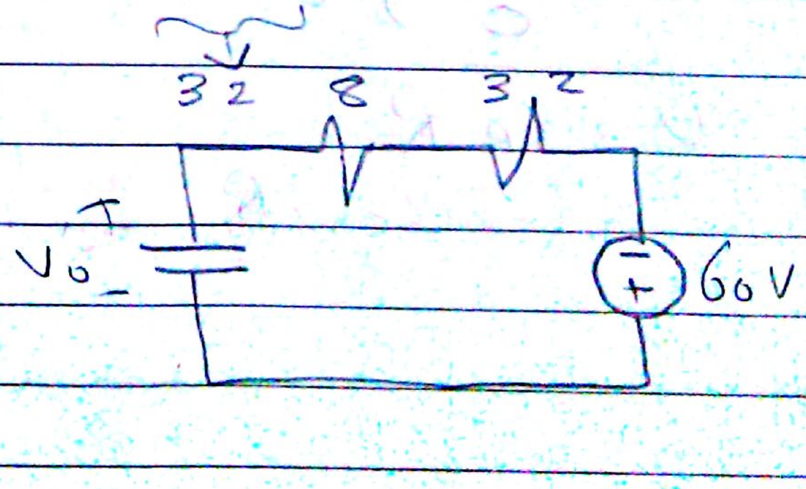


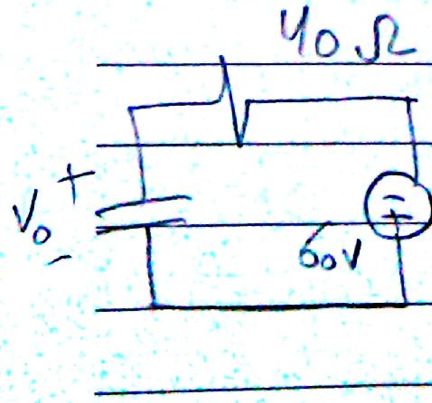
$$V_o = 40 * \frac{60}{80}$$

$$V_o = 30 \text{ Volt}$$





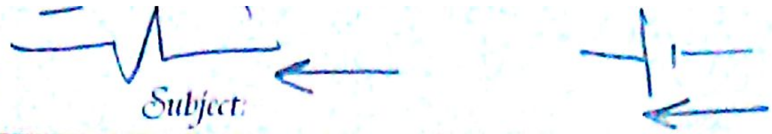


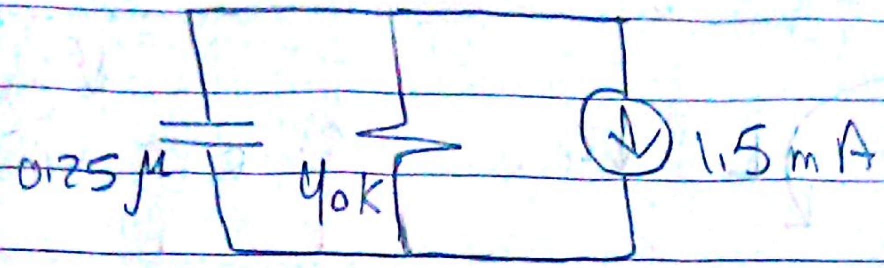


الجبار صاحب البلاوي

B.E

Date: Subject:





$\tau = 0.25 \times 40 \times 10^{-6} \times 10^{3} = 0.01\ s$

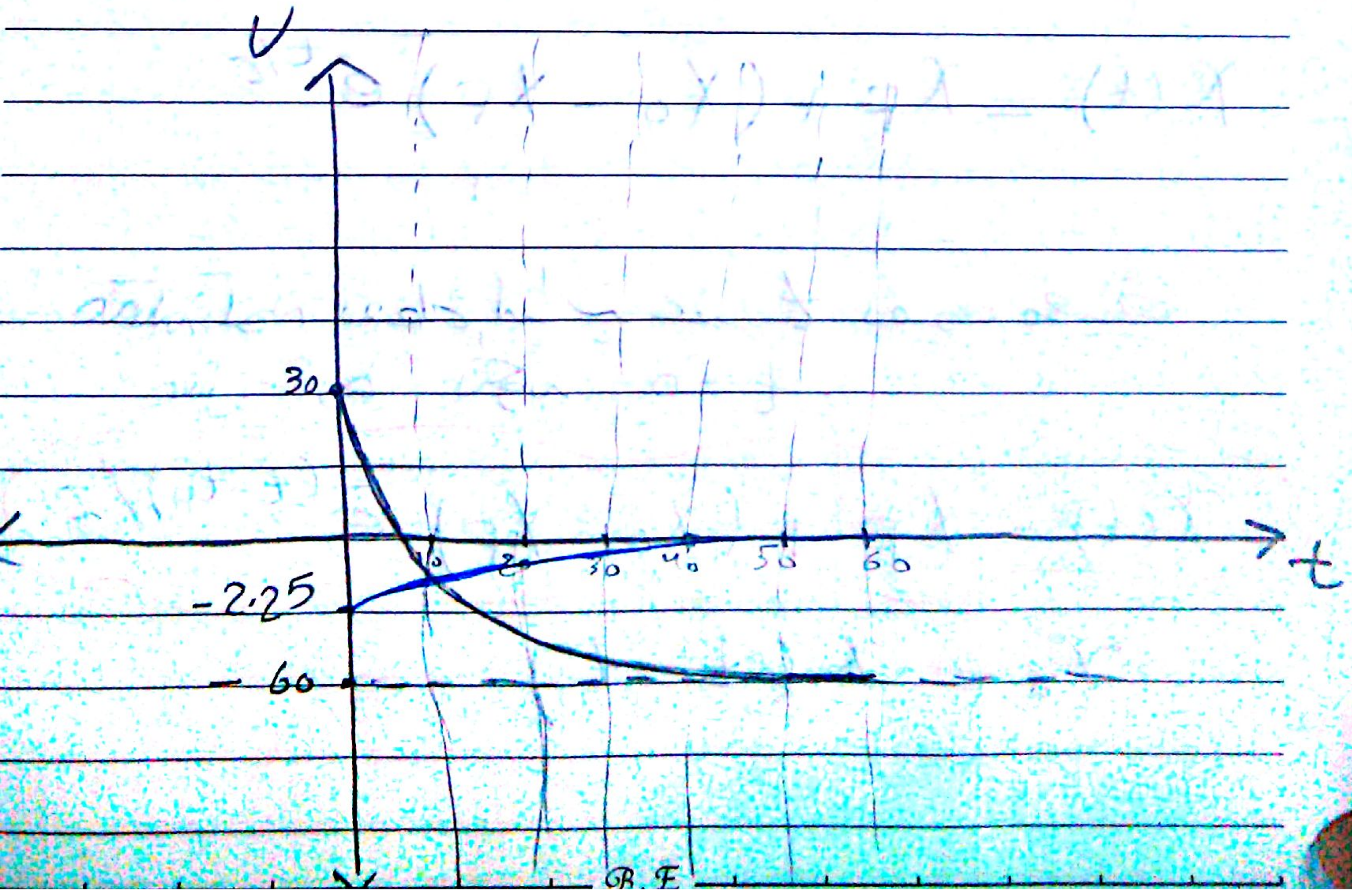
$V_f = I_s R$

$= -1.5\ m \times 40\ K = -60$

$V_0 = I_s R + (V_0 - I_s R)\, e^{-t/\tau}$

$V_0 = -60 + 90\, e^{-100t}$

$i_0 = -2.25\, e^{-100t}$ mA



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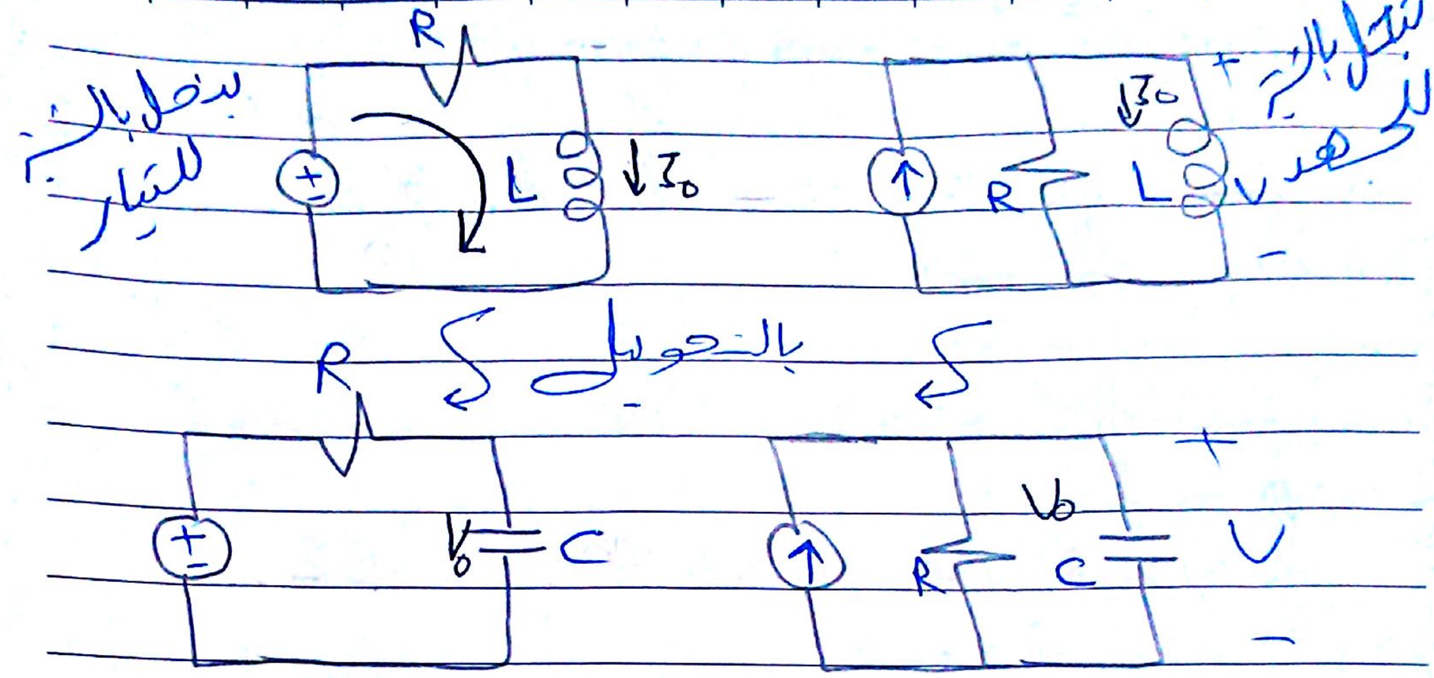


General Form قد تكون جهد او تيار

$$\frac{dX}{dt} + \frac{1}{\tau} X = K \rightarrow 0 \text{ \& step}$$

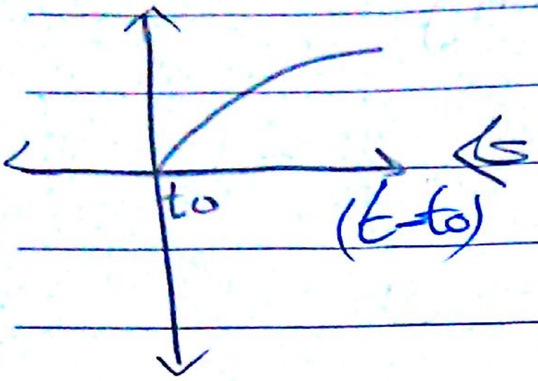
$$X(t) = X_F + (X_0 - X_F)\, e^{-t/\tau}$$

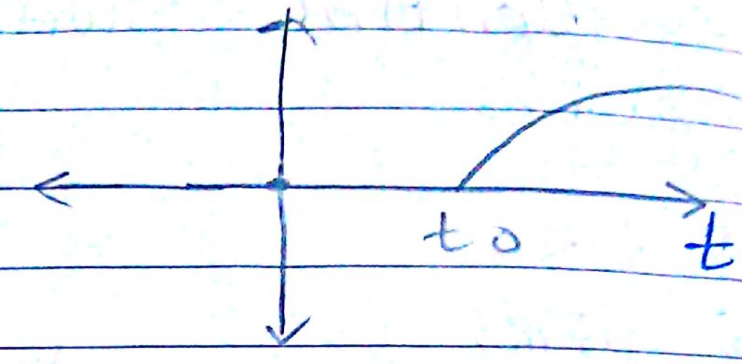
ممكن تكون نقطة البداية عند t بقيمة معينة
مش شرط تكون $t=0$.

$$\boxed{X(t) = X_F + (X_0 - X_F)\, e^{-(t-t_0)/\tau}}$$

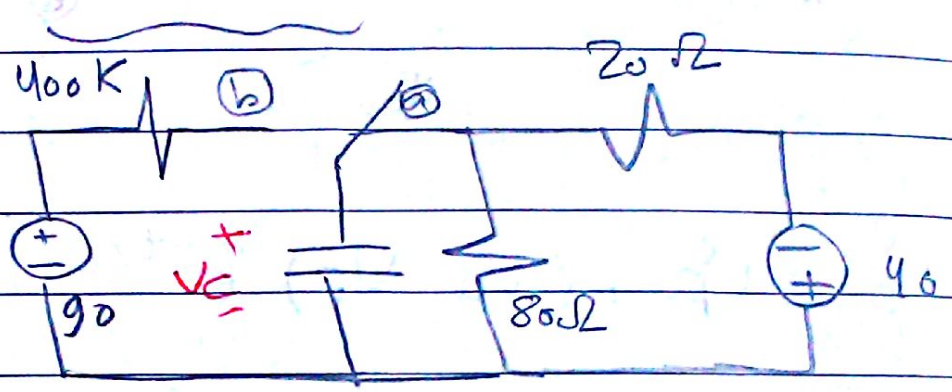
$$X_0 = X(t = t_0)$$

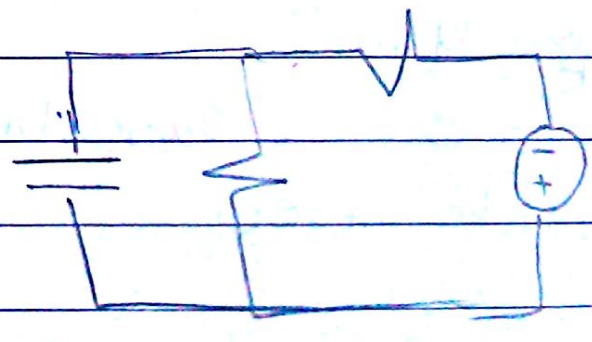
Date: Subject:





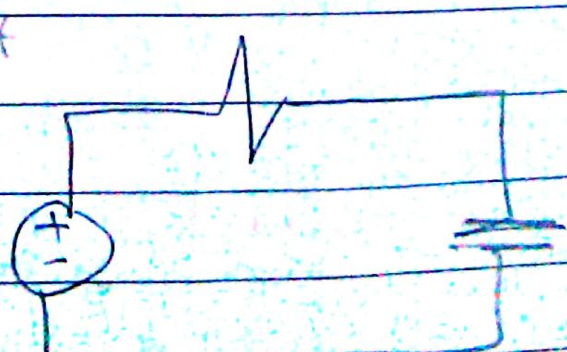
EM





$\tau = 0.25$

$V_C = 90 - 120e^{5t}$



Date: Subject: Lec 6

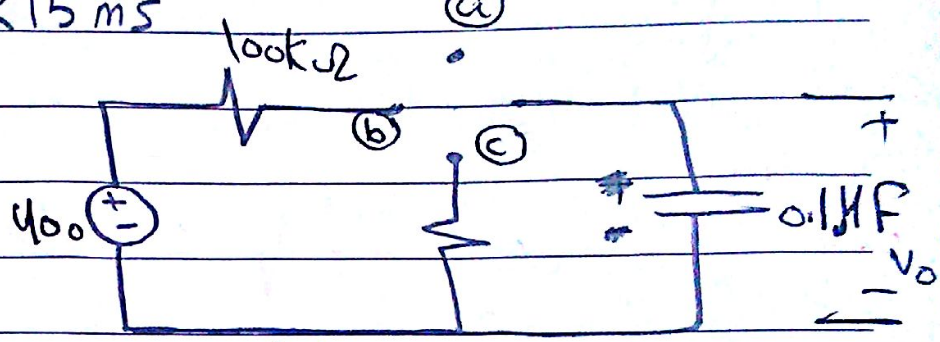
Sequential switching

تتابع

ⓐ $t < 0$

ⓑ ~~15 ms~~ $0 < t < 15\,ms$

ⓒ $15_{m} < t < \cdots$



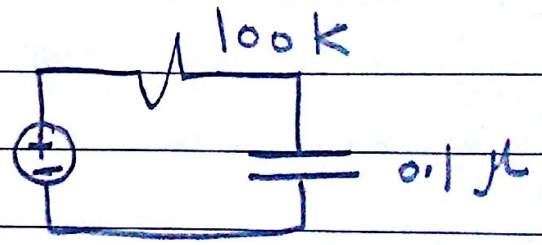
$$v_0 = v_f + (v_0 - v_f)\, e^{-t/\tau}$$

① 0 — 15 ms

$V_{f_1} = 400$

$V_{0_1} = 0 \rightarrow$ @ بأحوصلة

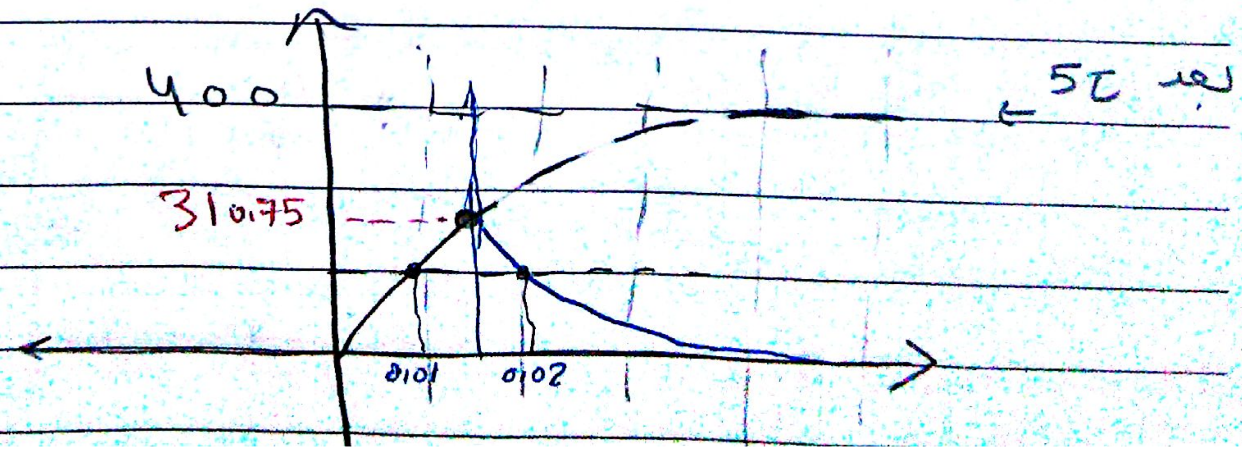
$\tau_1 = 10$ msec



① $v_{0_1}(t) = 400 - 400\, e^{-100 t}$

$= 400(1 - e^{-100 t})$

②



Date: Subject:

② $15ms \longrightarrow$

$V_{f2} = 0$

$V_{o2} = V_{o1}(t=15ms)$

$= 310.75V$

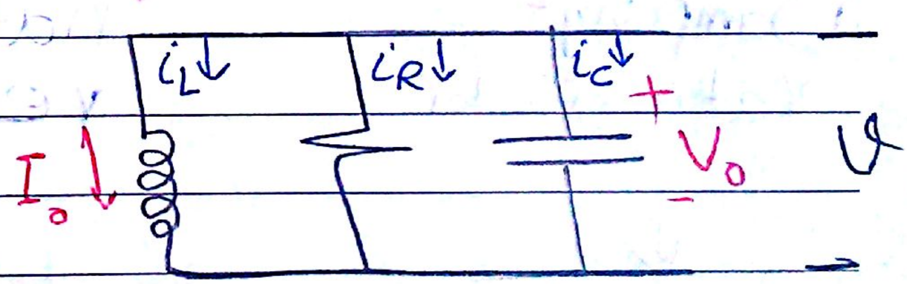
$\tau_2 = 5ms$

$$V_{o2}(t) = 310.75\, e^{-200(t-0.015)}$$

Response of second order system

1 Parallel RLC Circuit

مجموع التيارات = صفر



$$i_C + i_L + i_R = 0$$

$$C\frac{dv}{dt} + \frac{v}{R} + \frac{1}{L}\int v\,dt + I_o = 0$$

معادلة فيها تفاضل وتكامل

← في حالة Step هاضيف responce current source

← هادخل بدالة لجهد مشترك بينهم

Date: Subject:

$$C\frac{d^2v}{dt^2}+\frac{1}{R}\frac{dv}{dt}+\frac{1}{L}v=0$$

بقسمة الطرفين على C

$$\frac{d^2v}{dt^2}+\underbrace{\frac{1}{RC}}_{2\alpha}\frac{dv}{dt}+\underbrace{\frac{1}{LC}}_{\omega_0^2}v=0$$

معادلة تفاضلية خطية من الدرجة الثانية

$$\frac{d^2v}{dt^2}+2\alpha\frac{dv}{dt}+\omega_0^2 v=0$$

$$\boxed{S^2+2\alpha S+\omega_0^2=0}$$

α: Damping ratio & Factor

ω_0: natural Freq. resonant Freq $=\frac{1}{\sqrt{LC}}$

$$\alpha=\frac{1}{2RC}$$

$$S_1=-\alpha+\sqrt{\alpha^2-\omega_0^2}$$

$$S_2=-\alpha-\sqrt{\alpha^2-\omega_0^2}$$

$$\boxed{v=A_1e^{s_1t}+A_2e^{s_2t}}$$

ثوابت تكامل

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$v(0)$ —— (1) $\quad V(0) = A_1 + A_2$

$\frac{dv}{dt}(0)$ —— (2) $\quad \frac{dv}{dt}(0) = S_1 A_1 + S_2 A_2$

لو α اقل من W_0 ، لو α اكبر من W_0

$\alpha^2 < W_0^2$
under damped
responce ---

$\alpha^2 > W_0^2$
over damped
اخماد زائد.
responce ---

$\alpha^2 = W_0^2 \Rightarrow$ Critically damped
الاخماد الحرج.

$\alpha = \frac{1}{2RC} \quad W_0 = \frac{1}{\sqrt{LC}}$ } RLC

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Parallel RLC Natural

$\alpha = \frac{1}{2RC}$ rad/sec $\Big\{$ $W_0 = \frac{1}{\sqrt{LC}}$ rad/sec

$\alpha > W_0$ over damped
$\alpha = W_0$ Critical damped
$\alpha < W_0$ under damped

$$\boxed{v = A_1 e^{s_1 t} + A_2 e^{s_2 t}}$$

initial Condition يتم تصميم بـ ال A_1, A_2

① $v_{(0)} = V_0$

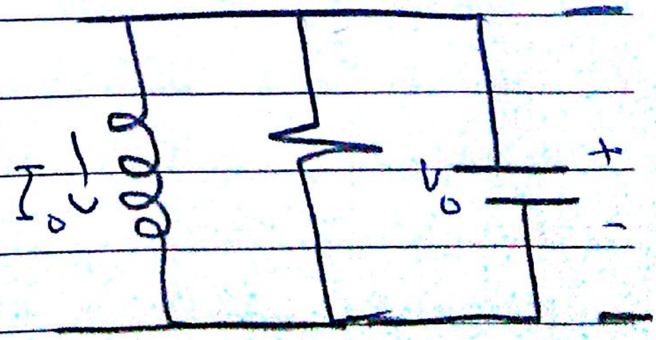
② $\frac{dv}{dt}(0)$ V/S

معدل تغير للتغير في الزمن في زمن $t=0$

نجري عملية التفاضل اولاً ثم نعوض عن $t=0$

$i_C = C\frac{dV}{dt}$

$\frac{dV}{dt} = \frac{1}{C} i_C$



KVL:

$i_L + i_R + i_C = 0$

$i_C = -(i_L + i_R)$

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$$\frac{dV}{dt} = -\frac{1}{C}(i_L + i_R) \rightarrow \text{general}$$

$$\frac{dv}{dt}(0) = -\frac{1}{C}\left[i_L(0) + i_R(0)\right]$$

$$= -\frac{1}{C}\left[I_0 + \frac{V_0}{R}\right]$$

$$V(0) = V_0 = A_1 + A_2$$

$$\frac{dv}{dt}(0) = -\frac{1}{C}\left[I_0 + \frac{V_0}{R}\right]$$

1) Over damped. Case

$$\boxed{\alpha > \omega_0}$$

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

مهم ⟸ S_2 , S_1 هايقوا رقم سالبة ..

→ negative real

$$V(t) = \underline{A_1}\, e^{\overset{-ve}{S_1} t} + \underline{A_2}\, e^{\overset{-ve}{S_2} t}$$

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Ex. 8.2

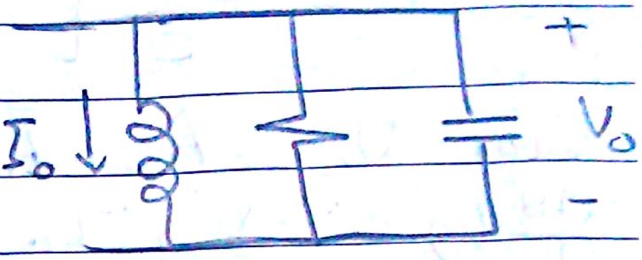
$L = 50\ mH$ — هكلم بقولي انه over damped

$C = 0.2\ \mu F$ — ويدين ال damped ratio

$R = 200\ \Omega$ — وبالتالي ال R سهل اجيبها

$V_0 = 12\ Volt$

$I_0 = 30\ mA$



① اجيب ω_0 , α

$$\alpha = \frac{1}{2RC} = 12500\ rad/s$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10000\ rad/s$$

$\because \alpha > \omega_0 \therefore \rightarrow$ over damped

② نحسب s_1 , s_2

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$
$$= -12500 + \sqrt{(12500)^2 - (10000)^2}$$
$$= -5000$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$
$$= -12500 - \sqrt{(12500)^2 - (10000)^2}$$
$$= -20000$$

③

$$V(t) = A_1 e^{-5000t} + A_2 e^{-20000t}$$

$$\frac{dV}{dt} = -5000 A_1 e^{-5000t} - 20000 A_2 e^{-2000...}$$

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$$\frac{dv}{dt}(0) = -5000A_1 - 20000A_2$$

$$V(0) = A_1 + A_2 = 12 \text{ Volt}$$

$$\frac{dv}{dt}(0) = -\frac{1}{C}\left[I_0 + \frac{V_0}{R}\right]$$

$$= -\frac{1}{0.2\mu f}\left[30mA + \frac{12V}{200}\right]$$

$$= -450\,000 \text{ Volt/sec}$$

$$A_1 + A_2 = 12 \rightarrow (1)$$

$$-5A_1 - 20A_2 = -450 \rightarrow (2)$$

نضرب الاولى 5 و نجمع

$$5A_1 + 5A_2 = 60$$

$$-5A_1 - 20A_2 = -450$$

$$-15A_2 = -390$$

$$A_2 = 26$$

$$A_1 = 12 - 26 = -14$$

$$\therefore V(t) = -14e^{-5000t} + 26e^{-20000t}$$

$$i_R(t) = \frac{V(t)}{R} = -0.07e^{-5000t} + 0.13e^{-20000t} \text{ A}$$

$$i_R(t) = -70e^{-5000t} + 130e^{-20000t} \text{ mA}$$

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$$i_C(t) = C\frac{dv}{dt}$$

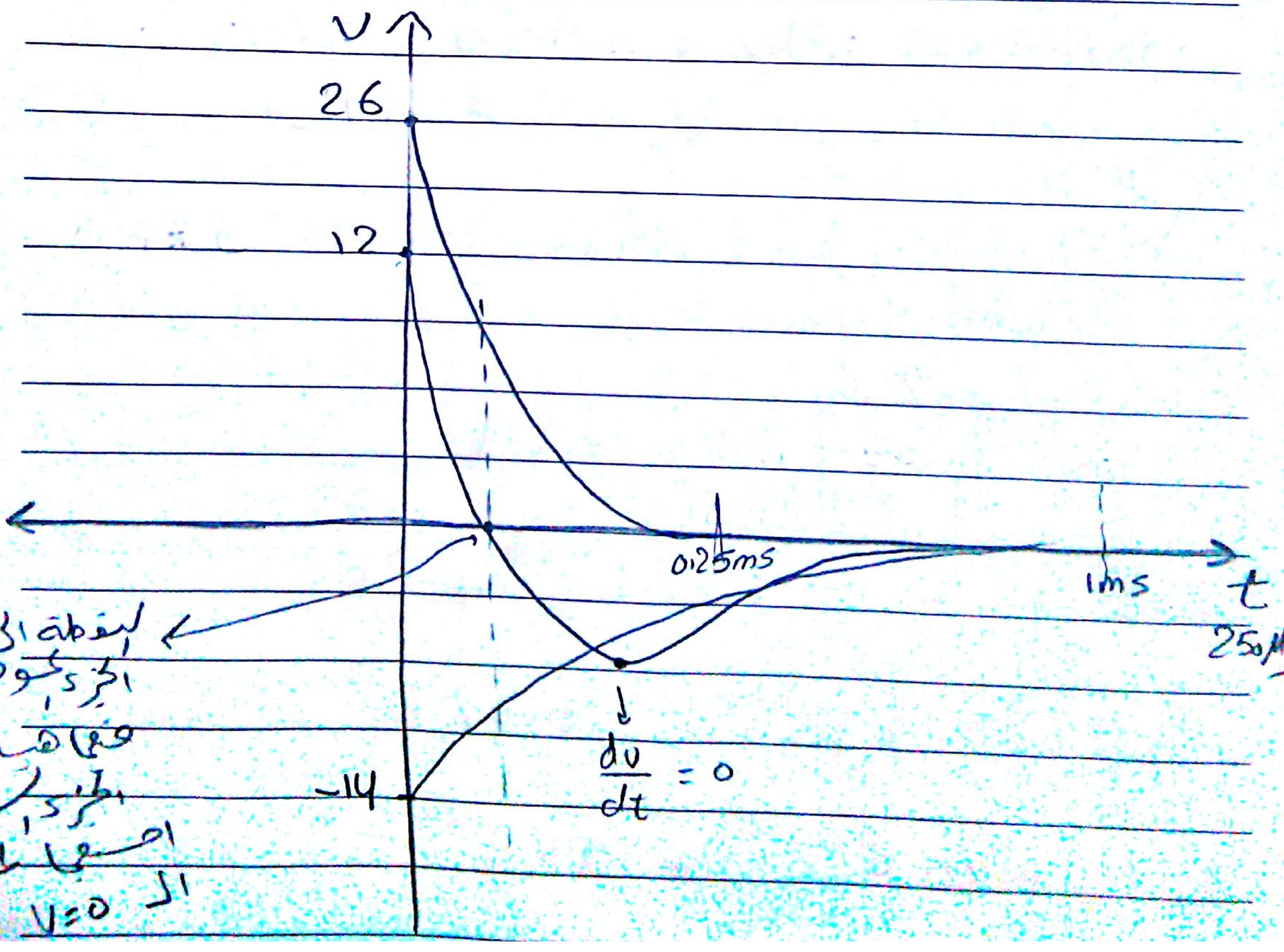
$$= 0.2\times10^{-6}\left[(-14)(-5000)e^{-5000t} + (26)\times(20000)e^{-20000t}\right]$$

$$= 0.014\,e^{-5000t} - 0.104\,e^{-20000t}\ A$$

$$= 14\,e^{-5000t} - 104\,e^{-20000t}\ mA$$

$$i_L = -i_R - i_C = 56\,e^{-5000t} - 26\,e^{-20000t}\ mA$$

$$\Rightarrow i_L = \frac{1}{L}\int v(t)\,dt + I_0$$



النقطة الي
الجزء الموجب
فيها صار
الجزء السالب
اصغر منا
ال $V=0$

Date: Subject:

ارسمها بالأمر لـ الحد الأول ← $V(t)$

لابد ل $\tau = \frac{1}{5000}$ فيكون ابطأ في الاضمحلال

وبينتهي عند $5\tau \leftarrow \frac{5}{5000} = \frac{1}{1000}$

$= 1 \times 10^{-3}$ sec $= 1$ msec

لكن الحد الثاني $5\tau = \frac{5}{20000}$

2.5×10^{-4} sec $= 0.25$ msec

يعني في حد هاينتهي في 1 msec والحد الثاني هاينتهي في 0.25 ms

المطلوب: ارسم 250 ms

at $v = 0$

$V(t) = 0$ at 41.27 ms

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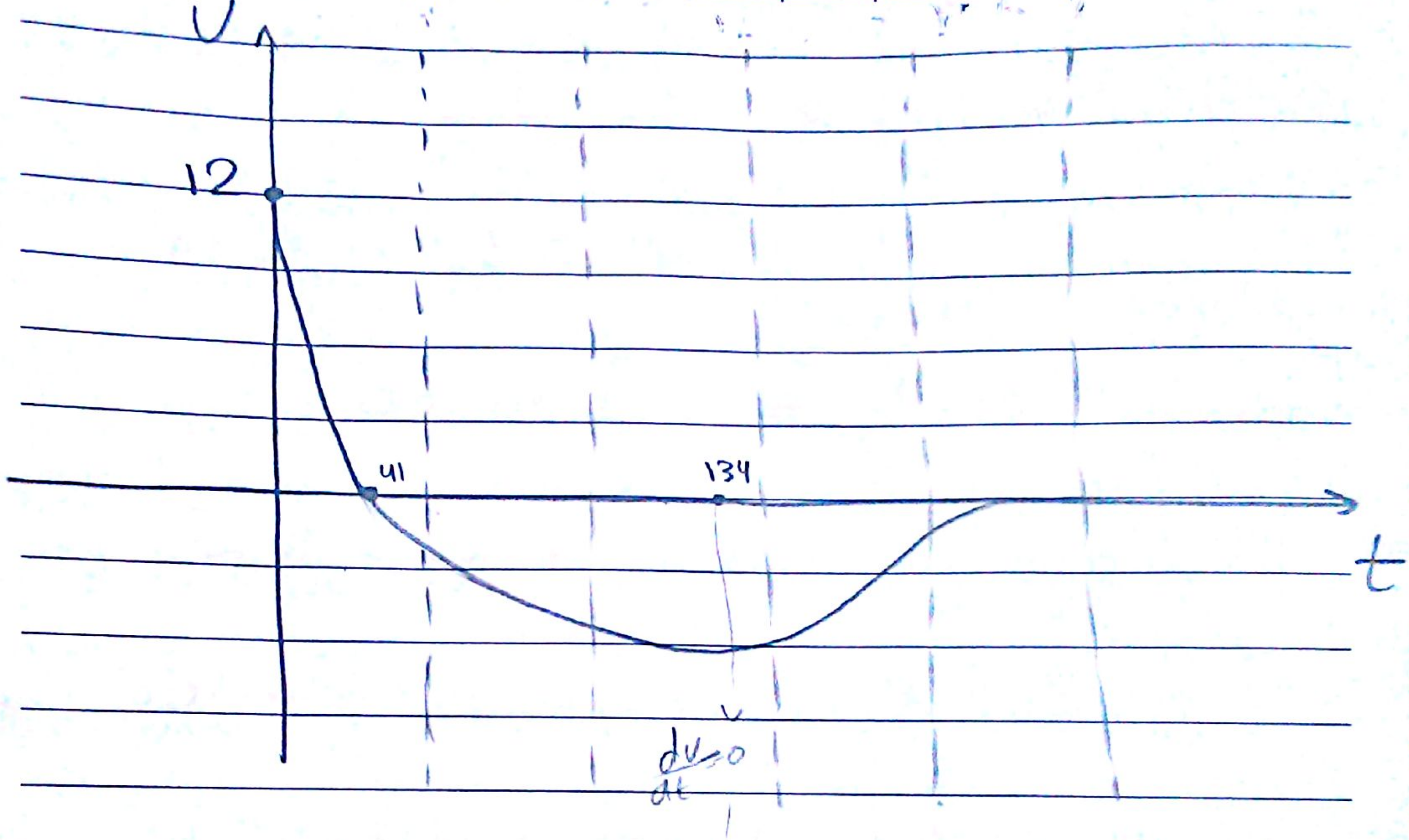

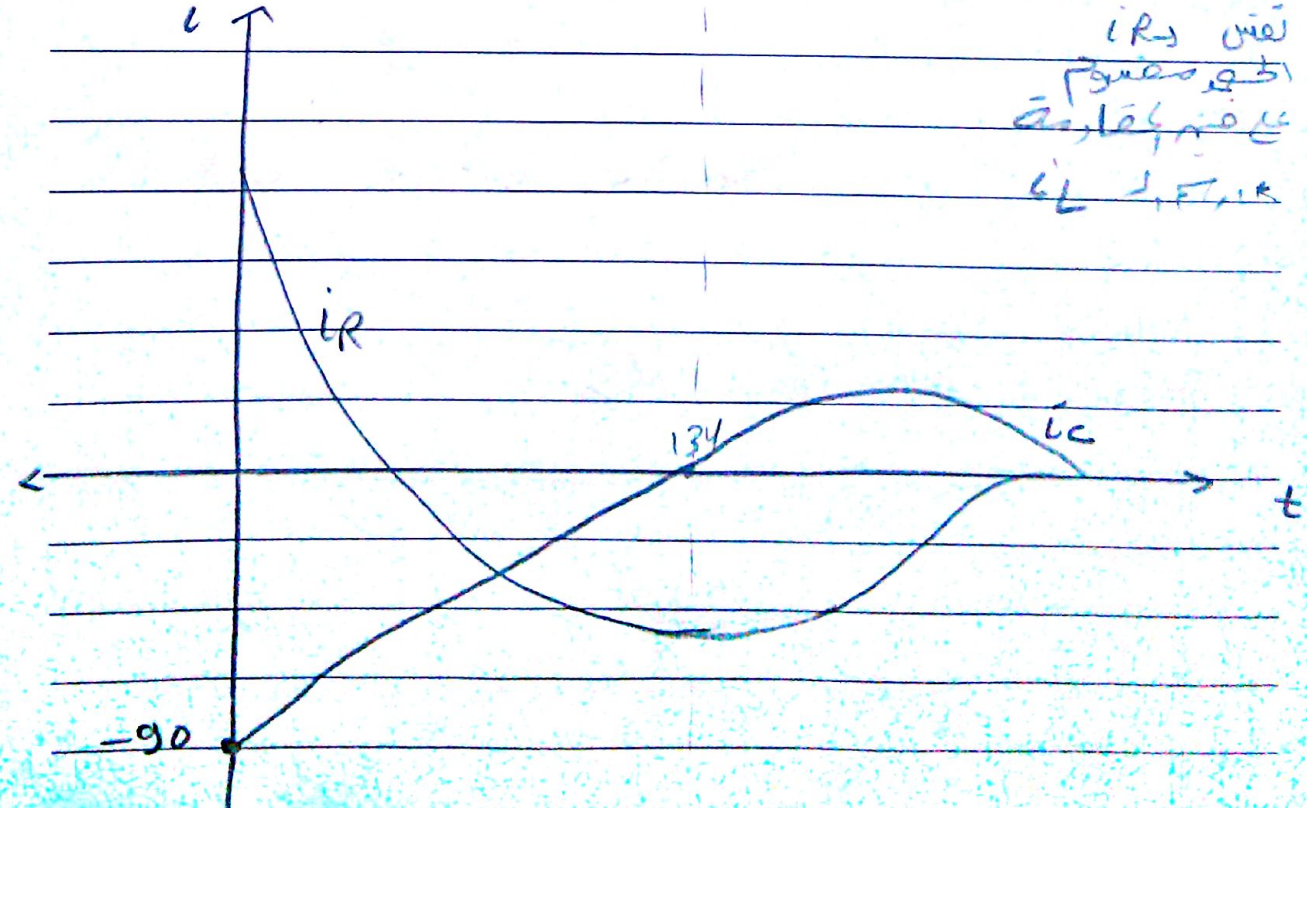

نفس الـ i_R
الحين مقسوم
ع قيم المقاومة
i_L لـ ١،٢،٣

Date:

Subject:

2 Under-Damped Case

$$\boxed{\alpha < \omega_0}$$

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$\alpha^2 - \omega_0^2 < 0$ $\xrightarrow{\text{طلع}}$ $j\sqrt{\omega_0^2 - \alpha^2}$

$\therefore$

$$S_1 = -\alpha + j\sqrt{\omega_0^2 - \alpha^2}$$

$$S_2 = -\alpha - j\sqrt{\omega_0^2 - \alpha^2}$$

الجزء real دايمًا -ve

$$\sqrt{\omega_0^2 - \alpha^2} \Longrightarrow \omega_d$$

ω_d: damped Frequency

ω_0: undamped natural Frequency

B. F

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$$V(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

Complex. ارقام مركبة ← s_1, s_2

$$V(t) = A_1 e^{(-\alpha + j w_d) t} + A_2 e^{(-\alpha - j w_d) t}$$

$$= A_1 e^{-\alpha t} e^{+j w_d t} + A_2 e^{-\alpha t} e^{-j w_d t}$$

$$= e^{-\alpha t} \left[A_1 e^{+j W_d t} + A_2 e^{-j W_d t} \right]$$

$$e^{\pm j\theta} = \cos\theta \pm j \sin\theta$$ Eular

$$V(t) = e^{-\alpha t} [A_1 \cos\theta + j A_1 \sin\theta + A_2 \cos\theta - j A_2 \sin\theta]$$

$$\theta = W_d t$$

$$\therefore V(t) = e^{-\alpha t} [\underline{A_1 \cos W_d t} + \underline{j A_1 \sin W_d t} + \underline{A_2 \cos W_d t} - \underline{j A_2 \sin W_d t}]$$

$$= e^{-\alpha t} [(A_1 \cos W_d t + A_2 \cos W_d t) + j(A_1 \sin W_d t - A_2 \sin W_d t)]$$

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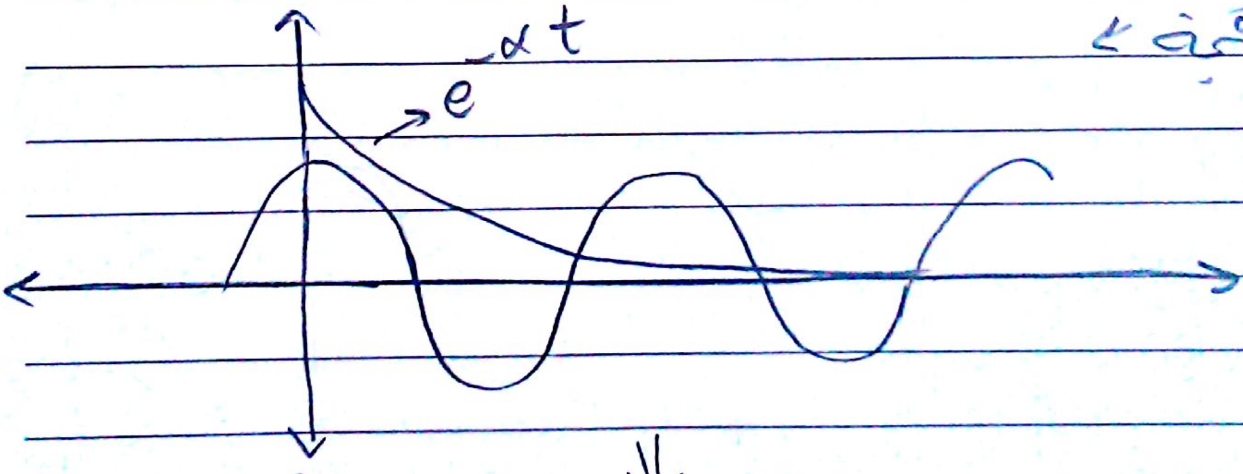
$$\therefore V(t) = e^{-\alpha t}\left[\underbrace{(A_1+A_2)}_{\text{real}} \cos w_d t + \underbrace{j(A_1-A_2)}_{\text{real}} \sin w_d t\right]$$

$$\boxed{V(t) = e^{-\alpha t}\left[B_1 \cos w_d t + B_2 \sin w_d t\right]}$$

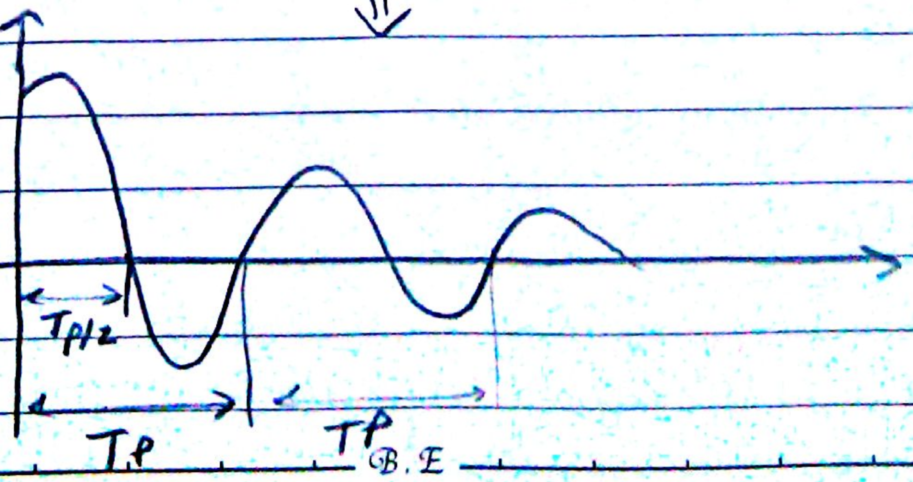
$$\frac{dV}{dt}(0) = -\alpha B_1 + W_d B_2$$

$$V(0) = B_1$$

لو B_2, B_1 ليهم قيمة ؟



$\Downarrow$



B.E